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D-branes and strings as non-commutative solitons

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Abstract: The non-commutative geometry of a large auxiliary B-field simplifies the construction of D-branes as solitons in open string field theory. Similarly, fundamental strings are constructed as localized flux tubes in the string field theory. Tensions are determined exactly using general properties of non-BPS branes, and the non-abelian structure of gauge fields on coincident D-branes is recovered.

Keywords: Solitons Monopoles and Instantons, D-branes, Non-Commutative Geometry.

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Contents

1. Introduction

General arguments [1, 2, 3], explicit calculations in truncated open string field theory [4, 5, 6, 7], and renormalization group analysis of relevant boundary perturbations [8] all suggest that D-branes can be constructed as solitons or lumps in open string field theory. Analogies between tachyon condensation in open string theory and confinement of electric charge [9] have also motivated suggestions that macroscopic closed strings can be described by flux tubes in open string field theory [4, 10, 11]. It is known that fundamental strings ending on D-branes can be viewed as flux tubes [12, 13, 14], the new element is the idea that such a description should be valid even in the absence of D-branes.

One problem in trying to construct these solutions explicitly is that they are string scale objects, and the string field theory action contains an infinite number of higher derivatives with coefficients set by the string scale. As a result it is difficult to obtain an accurate description of Dp-branes for small p, the non-abelian structure on multiple D-branes is far from obvious, and it has not been possible to obtain a concrete understanding of the flux tube solution.

In this paper we will show that these difficulties can be overcome by the introduction of non-commutative geometry via a background B-field [15, 16]. We consider solitons on the world-volume of an unstable D-brane in the presence of a large B-field. The solutions we study are the non-commutative solitons recently constructed by Gopakumar, Minwalla and Strominger (GMS) [17]. The solitons, though string scale with respect to the original space-time metric, are much larger than the string scale as measured by the effective open string metric of Seiberg and Witten [16]; it is this latter fact which facilitates the analysis. Far from the solitons the tachyon on the original unstable D-brane will have condensed to its local minimum, so that the soliton field configuration is asymptotic to the closed string vacuum without D-branes. The B-field is thus pure gauge far from the solitons, and the solutions represent D-branes and strings in the standard closed string vacuum.

A remarkable feature of the GMS solitons is that many of their properties are insensitive to details of the field theory to which they are solutions — in our case open string field theory on the unstable D-brane. Thus, for the most part we will consider only the low-energy dynamics of the light modes of the open string field. For most of our considerations we do not need to know the detailed shape of the tachyon potential, we require only the height of its local maximum. Happily, this is one of the few properties of unstable D-branes that we understand precisely, from a conjecture by Sen [1, 18]. Combining these observations therefore allows us to verify that the solitons have exactly the right tensions to be identified as D-branes. We also obtain the correct content of world-volume fields on the D-branes, and find the expected non-abelian gauge structure for multiple D-branes.

Classical open string field theory does not contain closed strings, they appear only as poles in loop diagrams. The arguments referred to earlier suggest however that after tachyon condensation the open string degrees of freedom are frozen out and one should find macroscopic closed strings in the classical theory. In the semi-classical limit these should be interpretable as solitons. Indeed, we find that the magic of non-commutativity facilitates a concrete construction where the fundamental string is identified with a flux tube. The fluctuations of the flux tube can be analyzed explicitly and, in a suitable approximation, are described by the Nambu-Goto action.

The paper is organized as follows. In section 2 we first review a few properties of non-commutative field theories and their solitons, we show how to embed this discussion in string theory via unstable D-branes, and then proceed to compute the tension of these solitons in string theory and identify them with D-branes. In section 3 we consider the massless gauge fields in the string field theory and see how these descend to the soliton world-volume. The gauge fields and their interplay with the tachyon field play an important role in obtaining the correct D-brane collective dynamics. In section 4 we extend the discussion to D-branes in type-II string theory. Finally, in section 5, we use similar methods to construct a flux tube in the open string field theory. We compute its tension and find its excitations, and are led to conclude that it can be identified with the closed fundamental string expected in the vacuum after tachyon condensation.

Notation: we will denote the complete set of space-time coordinates by x^μ, non-commutative directions by x^i, and the remaining directions by x^a.

2. Soliton solutions and their tensions

2.1 The non-commutative limit

We will be considering Minkowski space-time with closed string metric $g_{\mu\nu} = \eta_{\mu\nu}$ in the presence of a constant $B_{\mu\nu}$ field, the latter taking non-vanishing values only in purely spatial directions. As explained in [16], one should distinguish between the closed string metric $g_{\mu\nu}$ and the open string metric $G_{\mu\nu}$ which are related by

$$G_{\mu\nu} = g_{\mu\nu} - (2\pi\alpha')^2 (Bg^{-1}B)_{\mu\nu} \,. \tag{2.1}$$

The open string field theory action is related to the one without a B-field by using the metric $G_{\mu\nu}$, replacing ordinary products of fields by $\star$ products,

$$A(x)B(x) \to A \star B = e^{\frac{i}{2}\theta^{\mu\nu}\partial_\mu\partial_{\nu'}} A(x)B(x')|_{x=x'} \,, \tag{2.2}$$

and replacing the open string coupling g_s by

$$G_s = g_s \Big(\frac{\det G}{\det(g + 2\pi\alpha' B)} \Big)^{1/2} . \tag{2.3}$$

Here

$$\theta^{\mu\nu} = -(2\pi\alpha')^2 \left(\frac{1}{g + 2\pi\alpha' B} B \frac{1}{g - 2\pi\alpha' B} \right)^{\mu\nu} . \tag{2.4}$$

Now there is a new dimensionless parameter $\alpha' B_{\mu\nu}$, or equivalently $\theta^{\mu\nu}/\alpha'$, that can be varied in order to simplify the analysis. Denoting the directions in which $B_{\mu\nu}$ is non-vanishing as x^i, we will be interested in the limit of large non-commutativity, $\alpha' B_{ij} \to \infty$ with g_{ij} held fixed. In this limit the solitons will become much larger than the string scale when measured in the open string metric and this will lead to many simplifications. To avoid confusion, we note that there is an equivalent, but perhaps more familiar, form of the limit: $\alpha' B_{ij} \to 0$, $g_{ij} \to 0$, and G_{ij} is fixed. In this form one has $\theta^{ij}/\alpha' \to \infty$. The two versions of the limit are simply related by a coordinate transformation, $x^i \to 2\pi\alpha' B_{ij} x^j$. In either form of the limit,

$$\theta^{ij} = \left(\frac{1}{B} \right)^{ij} . \tag{2.5}$$

2.2 Solitons in string field theory

Let us review some aspects of D-branes as solitons in open string field theory. Our primary focus will be solitons on the world-volume of a bosonic D25-brane, although we will also discuss type-II D-branes.

The bosonic D25-brane has on its world-volume a tachyon, $m_t^2 = -1/\alpha'$, with a potential indicated schematically in figure 1. We keep an explicit factor of the D25-brane tension, T_{25}, in front of the action, so that the physical tachyon potential is $T_{25}V(t)$, and we have shifted the tachyon so that the local minimum is at $t = 0$. The unstable local maximum $t = t_*$ represents the space filling D25-brane, with $T_{25}V(t_*) = T_{25}$. The local minimum $t = 0$ is the closed string vacuum without D-branes, $V(0) = 0$.

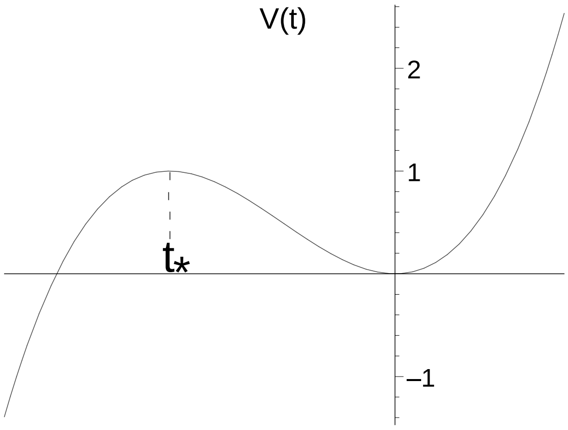

Figure 1: The bosonic open string tachyon potential.

The tachyon action supports unstable soliton solutions which are asymptotic to the closed string vacuum at $t = 0$, and it has been proposed to identify these with bosonic Dp-branes with $p < 25$. Such solitons have been constructed numerically in level truncated open string field theory in [4, 5], and for sufficiently large p good agreement was found between the tension and low lying spectra of the soliton and those of bosonic Dp-branes. However, the presence of higher derivatives in the string field theory action greatly complicates the analysis, and it seems very challenging to recover such fundamental properties as enhanced gauge symmetry for coincident D-branes.

In the present work we instead apply some powerful simplifications following from non-commutative geometry. Our starting point is an effective action for the tachyon obtained by integrating out (classically) all fields in the string field theory action which are "sourced" by the tachyon,

$$S = \frac{C}{g_s} \int d^{26}x \, \sqrt{g} \left(\frac{1}{2} f(t) g^{\mu\nu} \partial_\mu t \partial_\nu t + \cdots - V(t) \right) , \tag{2.6}$$

where $\cdots$ indicate higher derivative terms which will not be written explicitly, and we have explicitly displayed the string coupling by defining a g_s-independent constant

$$C = g_s T_{25} \, . \tag{2.7}$$

According to the conjecture of [2] the entire action vanishes at the local minimum: $f(0) = V(0) = 0$ (with corresponding equations for the higher derivative terms).

Now we turn on the B-field, which changes the action to

$$S = \frac{C}{G_s}\int d^{26}x\,\sqrt{G}\left(\frac{1}{2}f(t)G^{\mu\nu}\partial_\mu t\partial_\nu t + \cdots - V(t)\right), \tag{2.8}$$

where $\star$ products are now implied. Due to the non-commutativity one needs to specify an ordering of fields to define (2.8), but this level of precision will not be needed for the present analysis.

Soliton solutions in theories of this kind were constructed in [17] in the limit of large non-commutativity. A simple scaling computation shows that the potential term dominates over the derivative terms in this limit, so that the equation of motion for static solitons is

$$\frac{dV}{dt} = 0\,. \tag{2.9}$$

Localized soliton solutions to this equation exist due to the presence of the $\star$ product. The construction of GMS relies on the existence of functions ϕ satisfying[1]

$$\phi\star\phi = \phi\,, \tag{2.10}$$

since then

$$F(\lambda\phi) = F(\lambda)\phi\,, \tag{2.11}$$

for any function F of the form $F(x) = \sum_{n=1}^{\infty} a_n x^n$. In particular,

$$\left.\frac{dV}{dt}\right|_{t=\lambda\phi} = \left(\left.\frac{dV}{dt}\right|_{t=\lambda}\right)\phi\,, \tag{2.12}$$

and (2.9) is solved by choosing λ to be an extremum of V. Turning on B_{ij} in two directions, say $x_{1,2}$, the simplest function satisfying (2.10) is the gaussian

$$\phi_0(r) = 2e^{-r^2/\theta}\,, \qquad r^2 = x_1^2 + x_2^2\,, \tag{2.13}$$

with $B = B_{12}$, $\theta = 1/B$. For the potential indicated in figure 1 the solution will thus be

$$t = t_*\phi_0(r)\,. \tag{2.14}$$

Note that for this solution the tachyon asymptotically approaches its value $t = 0$ in the closed string vacuum. The resulting object is a $23+1$ dimensional soliton that we will identify with the D23-brane. The coordinate size of the soliton is $\Delta x \approx \sqrt{\theta} = 1/\sqrt{B}$, which goes to zero in the large B limit. However, for determining the importance of α' corrections the relevant quantity is $\Delta x_{\rm open} = \sqrt{G_{ij}\Delta x^i \Delta x^j} \approx \alpha'\sqrt{B}$. In the limit $\alpha' B \to \infty$ this is much larger than $\sqrt{\alpha'}$, so α' corrections, in the form of the derivative terms in (2.8), are suppressed.

The above construction easily generalizes to arbitrary even codimension solitons, for example by turning on equal B-fields in the $(12), (34), \ldots, (2q-1, 2q)$ planes and by replacing r^2 in (2.13) by $r^2 = x_1^2 + x_2^2 + \cdots + x_{2q}^2$. The resulting soliton is to be identified with a D$(25-2q)$-brane.

[1] Such functions have also made an appearance in the work of [19].

2.3 Tension of solitons

We now show that our solutions have the same tension as bosonic Dp-branes,

$$T_p = (2\pi)^{25-p}(\alpha')^{(25-p)/2}T_{25}\,. \tag{2.15}$$

We first consider the D23-brane soliton. At large non-commutativity we neglect the explicit transverse derivatives in (2.8) , and the action for translationally invariant configurations along the D23-brane is

$$S = -\frac{C}{G_s}\int d^{26}x\,\sqrt{G}V(t)\,. \tag{2.16}$$

Now we insert the soliton solution, use $V(t) = V(t_*)\phi_0(r)$, and integrate over x_1, x_2:

$$S = -\frac{CV(t_*)}{G_s}\int d^{24}x\int d^2x\,\sqrt{G}\phi_0(r) = -\frac{2\pi\theta CV(t_*)}{G_s}\int d^{24}x\,\sqrt{G}\,. \tag{2.17}$$

Next we use the relation (2.3) between G_s and g_s, which for large B-field is

$$G_s = \frac{g_s\sqrt{G}}{2\pi\alpha' B\sqrt{g}}\,. \tag{2.18}$$

In our conventions $V(t_*) = 1$ so, inserting this into (2.17) and using $\theta = 1/B$, we find

$$S = -(2\pi)^2\alpha'\frac{C}{g_s}\int d^{24}x\,\sqrt{g}\,. \tag{2.19}$$

Finally, recall $C = T_{25}g_s$. This identifies the tension of the soliton as

$$T_{\mathrm{sol}} = (2\pi)^2\alpha'\frac{C}{g_s} = (2\pi)^2\alpha' T_{25} = T_{23}\,. \tag{2.20}$$

Remarkably, in this limit we get precisely the correct answer without knowing the detailed form of the tachyon potential. We only need its value at the unstable extremum which follows from the conjecture of Sen, as substantiated in the work of [6, 3, 20, 8]. It is straightforward to generalize this result to arbitrary even codimension solitons, and to reproduce the formula (2.15) for odd p.

2.4 Multiple D-branes

There is a one-to-one correspondence between functions on the non-commutative R^2 transverse to the D23-brane, thought of as the phase space of a particle in one dimension, and operators acting on the Hilbert space of one-dimensional particle quantum mechanics. Multiplication by the $\star$ product goes over to operator multiplication, and integration over R^2 corresponds to tracing over the Hilbert space,

$$A\star B \leftrightarrow \hat{A}\hat{B}\,, \qquad \frac{1}{2\pi\theta}\int d^2x_i \leftrightarrow \mathrm{Tr}\,. \tag{2.21}$$

Under this correspondence, the equation (2.10) becomes the equation for a projection operator. This correspondence was utilized in [17] to construct more general soliton solutions. The soliton solution $t = t_* \phi_0$ (2.14) corresponds to the projection operator onto the ground state of a one-dimensional harmonic oscillator, $\phi_0 \sim |0\rangle\langle 0|$. Other solutions are obtained by choosing other projection operators, $\phi_n \sim |n\rangle\langle n|$, or we can choose a superposition (a level k solution in the terminology of GMS)

$$t_k = t_*(\phi_0 + \phi_1 + \cdots + \phi_{k-1})\,. \tag{2.22}$$

Since the projection operators are orthogonal, the energies just add

$$V(t_k) = kV(t_1)\,. \tag{2.23}$$

Thus this configuration corresponds to k coincident D-branes; further evidence for this claim will appear in succeeding sections.

Since the $\phi_m = |m\rangle\langle m|$ are a complete set of projection operators, the limit $k \to \infty$ of the level k solution is $t_\infty = t_* \mathbb{1}$. This solution can be identified with the D25-brane with no tachyon condensate; indeed, the tachyon takes the value $t = t_*$ everywhere, and the energy density is that of the D25-brane. At large k, the level k solution (2.22) represents, in the basis constructed in [17], a lump of size $r_k \sim \sqrt{k}$, which approximates the string field configuration of an unstable D25-brane for smaller radius, and the closed string vacuum state for larger radius. An amusing case is the projection operator complementary to (2.14), namely $t = t_*(1 - \phi_0)$. Evaluating the energy of this configuration, one formally finds the energy of a D25-brane 'minus' that of a D23.[2]

In the limit of infinite non-commutativity the action (2.8) can be written in operator form as

$$S = T_{23} \int d^{24}x_a \,\mathrm{Tr}\left(\frac{1}{2} f(\hat{t}) G^{ab} \partial_a \hat{t} \partial_b \hat{t} - V(\hat{t})\right), \tag{2.24}$$

where x^a are the commutative directions. The action in operator form has a manifest $\mathrm{U}(\infty)$ symmetry

$$\hat{t} \to U \hat{t} U^\dagger\,. \tag{2.25}$$

These group operations are familiar from the construction of the Matrix theory membrane [21, 22]: they correspond to area preserving diffeomorphisms. This is no accident. The Matrix theory membrane in non-compact space is essentially a D2-brane that has been bound to an infinite number of D0-branes such that the D0 charge density is finite (represented by a magnetic flux F). This charge density is equivalent to the B-flux of non-commutative geometry, since B and F are indistinguishable on

[2] Of course, the energy of the D25 is infinitely larger than that of the D23; to make this statement precise, one must go to finite volume, e.g. by compactifying the system on a large torus.

the brane. Thus the two constructions are identical, and it is useful to keep this relationship in mind. The main difference between the two situations is the presence of the tachyon field.

It is clear that, if we neglect the standard kinetic term, then acting with an area preserving diffeomorphism on a given configuration is a symmetry of (2.8) since it preserves the $\star$ product (which is defined in terms of the volume two-form) and the integration measure. On the other hand, the standard kinetic term involves also the metric g_{ij}. Only the elements of $\mathrm{U}(\infty)$ corresponding to translations and rotations preserve this metric and so are exact symmetries of the action. Thus, for purely scalar actions, as considered by GMS, the $\mathrm{U}(\infty)$ is an approximate global symmetry that becomes exact in the limit of infinite non-commutativity.

Solutions of the form (2.22) preserve a $\mathrm{U}(k)$ subgroup of the $\mathrm{U}(\infty)$ global symmetry of the action at infinite θ (times an irrelevant group acting in the orthogonal space), and all excitations on the branes will transform in multiplets of $\mathrm{U}(k)$. The breaking of $\mathrm{U}(\infty)$ to $\mathrm{U}(k)$ leads to an infinite number of Nambu-Goldstone bosons in the spectrum of the soliton. Again, for a purely scalar theory, these become pseudo-Nambu-Goldstone bosons at finite θ. As we discuss in section 3, the story changes after coupling to the gauge fields on the D-brane.

2.5 Tachyon on the soliton

The D25-brane tachyon reflects the fact that $t = t_*$ is an unstable point of the potential:

$$V(t) = V(t_*) + \frac{1}{2}V''(t_*)(t - t_*)^2 + \cdots , \qquad V''(t_*) < 0 , \tag{2.26}$$

and from (2.6) the mass of the open string tachyon is

$$m_t^2 = \frac{V''(t_*)}{f(t_*)} = -\frac{1}{\alpha'} . \tag{2.27}$$

The lower D-branes also have a tachyon with this mass, and we should recover it by studying fluctuations around the soliton.

As usual, we first consider the simplest case of two non-commutative directions x^i. Using the operator correspondence, a complete set of functions on this space is given by

$$\phi_{mn}(x^i) \sim |m\rangle\langle n| , \tag{2.28}$$

so the general fluctuation is

$$t + \delta t(x^\mu) = t_*\phi_{00}(x^i) + \sum_{m,n=0}^{\infty} \delta t_{mn}(x^a)\phi_{mn}(x^i) . \tag{2.29}$$

Reality of t requires δt_{mn} to be a hermitean matrix. As we have discussed, the fluctuations include Nambu-Goldstone bosons from the spontaneous breaking of $\mathrm{U}(\infty)$

by the soliton. As in GMS, the generators of $\mathrm{U}(\infty)$ are $R_{mn} = |m\rangle\langle n| + |n\rangle\langle m|$ and $S_{mn} = i(|m\rangle\langle n| - |n\rangle\langle m|)$, and the Nambu-Goldstone bosons are the nonzero components of $\delta t = [R_{mn}, t], [S_{mn}, t]$. We will see in the next section that in the D-brane application of interest $\mathrm{U}(\infty)$ is a gauge symmetry and the Nambu-Goldstone bosons are eaten by the Higgs mechanism. Thus we use the $\mathrm{U}(\infty)$ symmetry to set

$$\delta t_{0m} = \delta t_{m0} = 0\,, \qquad m \geq 1\,. \tag{2.30}$$

Substituting into the action and using the fact that $\phi_0 \equiv \phi_{00}$ is orthogonal to ϕ_{mn} for $m, n > 1$ we are left with the single tachyon fluctuation δt_{00} which we rename δt. We now use

$$V(t + \delta t \phi_0) = V(t_* + \delta t)\phi_0 = \left(V(t_*) + \frac{1}{2} V''(t_*)(\delta t)^2 + \cdots \right) \phi_0\,. \tag{2.31}$$

After integrating over the soliton the quadratic effective action for δt becomes

$$S = T_{23} \int d^{24}x_a \left(\frac{1}{2} f(t_*) \partial^a \delta t \partial_a \delta t - \frac{1}{2} V''(t_*)(\delta t)^2 \right)\,, \tag{2.32}$$

so the D23-brane correctly inherits the tachyon of the D25-brane,

$$m_t^2 = \frac{V''(t_*)}{f(t_*)} = -\frac{1}{\alpha'}. \tag{2.33}$$

This analysis can be easily repeated to obtain a tachyon on the more general solution (2.22); then acting with $\mathrm{U}(k)$ on this tachyon produces the expected k^2 tachyons. A notable point is that the general tachyon will contain operators of the form $|m\rangle\langle n|$, $m \neq n$. These correspond to non-spherically symmetric tachyon fluctuations in the transverse space. So the full k^2 of tachyons comes from including both spherically symmetric and non-symmetric tachyon configurations.

3. Coupling to gauge fields

A characteristic property of D-branes is the presence of gauge fields on their world-volume. In this section, we demonstrate that there exists a single massless gauge field on the non-commutative soliton, therefore providing further evidence that the soliton can be identified as a D-brane. Furthermore, when we generalize to level k solutions such as (2.22), the gauge symmetry is enhanced to $\mathrm{U}(k)$ in the appropriate way, with the components of the gauge fields transverse to the soliton behaving as the standard adjoint Higgs fields on a D-brane. Finally, the gauge symmetry removes from the spectrum the unwanted (pseudo) Nambu-Goldstone bosons describing soft deformations (2.25) of the soliton (2.14); they are eaten by the Higgs mechanism.

This last point brings out an intriguing aspect of our situation. Ordinarily, when a global symmetry is explicitly broken, no matter how softly, it cannot be gauged.

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One might then wonder how the U(∞) symmetry (2.25) can be gauged, since it is broken by the tachyon kinetic term at finite θ. The resolution of this puzzle is strikingly reminiscent of general relativity. There, the potential energy term in the action of a scalar field is invariant under volume-preserving diffeomorphisms; but the kinetic energy term is not. Rather, the coordinate transformations are gauged by coupling to a metric, without there being a corresponding global symmetry in the absence of gauging. We will see that, once we include the gauge fields, the approximate U(∞) global symmetry is realized as an exact gauge symmetry. In the present context, the non-commutative D-brane gauge field takes over the role of the metric in covariantizing area-preserving diffeomorphisms!

A note on conventions: in this section we often set $2\pi\alpha' = 1$ to avoid clutter.

3.1 The gauge theory and its symmetries

First consider the action for the D25-brane. Imagine integrating out all fields except for the tachyon and the gauge field. The result will be some gauge invariant expression involving an infinite number of derivatives and an infinite number of higher powers of the fields. Working in terms of the $\star$ product implies that the gauge transformation law of the non-commutative U(1) gauge field is

$$\delta_\lambda A_\mu = \partial_\mu \lambda - i[A_\mu, \lambda]\,, \tag{3.1}$$

where

$$[A_\mu, \lambda] = A_\mu \star \lambda - \lambda \star A_\mu\,. \tag{3.2}$$

The corresponding field strength is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]\,. \tag{3.3}$$

We also need to know the gauge transformation of the tachyon. String theory disk diagrams reveal that the tachyon transforms in the adjoint of the non-commutative U(1) [23], i.e.

$$\delta_\lambda t = -i[t, \lambda]\,, \tag{3.4}$$

with corresponding covariant derivative

$$D_\mu t = \partial_\mu t - i[A_\mu, t]\,. \tag{3.5}$$

We will restrict attention to terms quadratic in the field strength. The full action contains higher derivative terms in the commutative directions, but for clarity's sake these will not be displayed explicitly. The action is then,

$$S = \frac{C}{G_s} \int d^{26}x \sqrt{G} \left(\frac{1}{2} f(t) D^\mu t D_\mu t - V(t) - \frac{1}{4} h(t) F^{\mu\nu} F_{\mu\nu} \right). \tag{3.6}$$

As with the tachyon kinetic term, the gauge kinetic term $h(t)$ vanishes at the local minimum $t = 0$ according to the conjecture of Sen. A simple RG flow argument for this result is given in [8].

The derivatives in non-commutative directions are all suppressed by α', and so can be dropped in the large B limit, both in the action and in the gauge transformation laws. We again take the B-field to be non-vanishing in two directions x^i and denote the remaining coordinates by x^a. Expanding out the action in the large B limit yields

$$S = \frac{C}{G_s}\int d^{26}x\,\sqrt{G}\Big(\frac{1}{2}f(t)D^a t D_a t - \frac{1}{2}h(t)D^a A_i D_a A^i - V(t) - \frac{1}{2}f(t)[A_i, t][A^i, t] + \\ + \frac{1}{4}h(t)[A_i, A_j][A^i, A^j] - \frac{1}{4}h(t)F^{ab}F_{ab}\Big)\,. \tag{3.7}$$

The A_i now appear as scalar fields. The action in this limit is invariant under the gauge transformations

$$\begin{aligned} \delta_\lambda t &= -i[t, \lambda]\,, \\ \delta_\lambda A_i &= -i[A_i, \lambda]\,, \\ \delta_\lambda A_a &= \partial_a \lambda - i[A_a, \lambda]\,. \end{aligned} \tag{3.8}$$

By passing to the operator description in which $\frac{1}{2\pi\theta}\int d^2x \to \mathrm{Tr}$ and fields are replaced by matrix representations of Hilbert space operators, one sees that (3.7) is a $23+1$ dimensional $\mathrm{U}(\infty)$ gauge theory coupled to the adjoint scalars t, A_i. This $23+1$ gauge theory emerges even before considering the soliton background. Note that the transformation law of the tachyon is just the infinitesimal form of (2.25); the $\mathrm{U}(\infty)$ symmetry is a gauge symmetry, as advertised previously. At finite non-commutativity the term $\partial_i\lambda$ in $\delta_\lambda A_i$ should be restored; the gauge symmetry then remains exact.

Before studying the soliton solution and its fluctuations, we briefly digress to explain the emergence of the $23+1$ dimensional $\mathrm{U}(\infty)$ gauge theory from another viewpoint (besides the relation to Matrix theory given above), essentially repeating comments in [16]. The open string theory effective action at lowest order is given by the path integral on the disk Σ with boundary conditions

$$g_{ij}\partial_n x^j + B_{ij}\partial_t x^j|_{\partial\Sigma} = 0\,. \tag{3.9}$$

For $B = 0$ one has Neumann boundary conditions, $\partial_n x^i = 0$, while for $\alpha' B \to \infty$ one finds Dirichlet boundary conditions, $\partial_t x^i = 0$. In the large B limit one thus finds D23-branes, though located at arbitrary transverse positions. In fact, this can be thought of as a continuous distribution of D23-branes with density proportional to B (see for example [24, 25]). In the large B limit, the theory governing such a system would be a $23+1$ dimensional $\mathrm{U}(\infty)$ gauge theory, which is in accord with the result above.

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3.2 Tachyon-gauge field fluctuations about the soliton

We now consider fluctuations of the action (3.7) around the soliton solution

$$t = t_* \phi_{00} \,. \tag{3.10}$$

The soliton breaks the U(∞) gauge symmetry down to U(1) (times the group "U($\infty-$1)" which will play no role in the discussion.) Working in unitary gauge we can take the tachyon fluctuations as in (2.29) and (2.30). The fluctuations of the gauge field are

$$\begin{aligned} A_a(x^\mu) &= \sum_{m,n=0}^{\infty} A_a^{mn}(x^b)\phi_{mn}(x^i)\,, \\ A_i(x^\mu) &= \sum_{m,n=0}^{\infty} A_i^{mn}(x^a)\phi_{mn}(x^j)\,, \end{aligned} \tag{3.11}$$

where A_a^{mn} and A_i^{mn} are hermitean as matrices with indices mn. Inserting the fluctuations into the action (3.7) we find that all modes with $m, n > 0$ are projected out by the soliton background, as was seen previously for the pure tachyon fluctuations. Remaining are δt_{00}, A_i^{00}, A_a^{00}, as well as the $0m$ and $m0$ components of A_i, A_a. For convenience we drop the explicit 00 indices on the first three fields; after integrating over the transverse space their action is found to be

$$S = T_{23}\int d^{24}x\sqrt{g}\left(\frac{1}{2}f(t_*)\partial^a\delta t\partial_a\delta t - V(t_*+\delta t) + \frac{1}{2}h(t_*)\partial^a A_i\partial_a A_i - \frac{1}{4}h(t_*)F^{ab}F_{ab}\right), \tag{3.12}$$

where $F_{ab} = \partial_a A_b - \partial_b A_a$ is the standard field strength. This is precisely the right action to describe the tachyon, gauge field, and transverse scalars on a D23-brane. The transverse scalars A_i play the role of translational collective coordinate, and are guaranteed to be massless by the original non-commutative U(1) gauge invariance.

The corresponding story for the $0m$ and $m0$ components of A_i, A_a involves some additional subtleties. For A_i we find the action,

$$S = T_{23}\sum_{m=1}^{\infty}\int d^{24}x\,\sqrt{g}\left(\frac{1}{2}h(t_*)\partial^a A_i^{0m}\partial_a A_i^{m0} - \frac{1}{2}t_*^2 f(t_*)A_i^{0m}A_i^{m0}\right). \tag{3.13}$$

A very similar result holds for A_a. The second term appears to be the standard mass term for W-bosons after gauge symmetry breaking, giving a mass

$$m_A^2 = \left(\frac{t_*}{2\pi\alpha'}\right)^2 . \tag{3.14}$$

We have inserted for definiteness the perturbative values $h(t_*) = (2\pi\alpha')^2, f(t_*) = 1$. The appearance of the tachyon VEV t_* is interesting because it is the only point in this work where we need a quantity that is not known exactly. It is possible to estimate t_* in the level-truncation scheme. This yields an expression $t_*^2 \sim \alpha'$ with a coefficient that rapidly converges to some value of $\mathcal{O}(1)$ [6, 3, 20].

Although these states have mass of order the string scale and are thus removed from the low energy spectrum, we are naively left with infinitely many massive modes on the soliton, degenerate as $\theta \to \infty$ and with a spacing proportional to $1/\sqrt{\theta}$ at finite θ, in clear contradiction with the known spectrum of D-branes.

We believe the resolution of this puzzle involves the higher derivative terms in the commutative directions and the freezing out of the open string degrees of freedom in the closed string vacuum. For example, Kostelecky and Samuel showed in [6] that non-local terms in the string field theory action which appear due to the substitution

$$\phi \to \tilde{\phi} \equiv e^{\alpha' \ln 3\sqrt{3}/4 \partial_\mu \partial^\mu} \phi \tag{3.15}$$

in cubic interaction terms, have the effect of modifying the tachyon propagator in the presence of tachyon condensation so that there is no physical pole.

It is reasonable to expect that, in our context, the apparent infinite tower of massive gauge fields on the soliton is similarly removed. It would be nice to justify this expectation with an explicit computation, but we have not yet done so.

We should also point out that the higher derivative terms do not affect our previous calculations in any substantial way. By writing out the higher derivative terms explicitly and repeating our analysis one sees that the D23-brane precisely inherits the higher derivative terms with the same coefficients as on the D25-brane. So for the modes considered previously, whatever the higher derivative terms do to the spectrum on the D25-brane, they do the same thing on the D23-brane.

3.3 Multiple D-branes

Solitons with the tachyon profile given in (2.22) are interpreted as k coincident D23-branes, and we expect to see the action (3.12) replaced by an action with U(k) gauge invariance. First consider the field strength F_{ab}. Inserting the expansion (3.11) into (3.3) we find

$$F_{ab} = F_{ab}^{mn} \phi_{mn} \,, \tag{3.16}$$

with

$$F_{ab}^{mn} = \partial_a A_b^{mn} - \partial_b A_a^{mn} - i[A_a, A_b]^{mn} \,. \tag{3.17}$$

In the latter equation A_a are being multiplied as matrices. We can now work out the final term in (3.7) for the background (2.22) as

$$\frac{C}{G_s} \int d^{26}x \sqrt{G} \left(-\frac{1}{4} h(t_k) F^{ab} F_{ab} \right) = T_{23} \int d^{24}x \sqrt{g} \left(-\frac{1}{4} \sum_{m=0}^{k-1} \sum_{n=0}^{\infty} F^{ab\, mn} F_{ab}^{nm} \right) . \tag{3.18}$$

Components of the gauge field A_a^{mn} with $m, n > k-1$ have been projected out by the soliton background. There still remain in the action (3.18) components with $m < k$, $n \geq k$ or $m \geq k$, $n < k$, but we conjecture that these states are removed from the spectrum by the mechanism described in the previous subsection. What remains then is precisely the gauge kinetic term for a $\mathrm{U}(k)$ gauge theory.

Similarly including t and A_i is straightforward. Ordinary derivatives are replaced by $\mathrm{U}(k)$ covariant derivatives, and altogether we recover the $\mathrm{U}(k)$ gauge invariant version of (3.12). This enhancement of the gauge symmetry for coincident solitons, though following rather easily from the present formalism, is highly nontrivial from a broader field theory vantage point, and provides strong evidence for the identification of the solitons with D-branes.

3.4 Massive modes

Though we have so far focussed on the tachyon and gauge field fluctuations, it is not hard to generalize our considerations to show that the solitons correctly inherit from the D25-brane the entire tower of open string states. Consider for illustration the quadratic terms for some massive field ψ,

$$S = \frac{C}{G_s} \int d^{26}x \sqrt{G} \left(\frac{1}{2} f(\Phi) \partial_\mu \psi \partial^\mu \psi - \frac{1}{2} m^2(\Phi) \psi^2 \right), \tag{3.19}$$

where Φ denotes all fields in the theory. The mass of ψ on the D25-brane is determined by evaluating $f(\Phi)$ and $m(\Phi)$ at the local maximum $\Phi = \Phi_*$. The soliton background corresponds to $\Phi = \Phi_* \phi_0$, and we consider the fluctuations $\psi = \sum \psi_{mn} \phi_{mn}$. As before, $m, n \neq 0$ fluctuations are projected out; $m = 0$, $n > 0$ and $m > 0$, $n = 0$ fluctuations are removed (we conjecture) by higher derivative terms; so what remains, after integrating over the soliton, is

$$S = T_{23} \int d^{24}x \left(\frac{1}{2} f(\Phi_*) \partial_\mu \psi_{00} \partial^\mu \psi_{00} - \frac{1}{2} m^2(\Phi_*) \psi_{00}^2 \right). \tag{3.20}$$

Since what appears in the action are the functions $f(\Phi)$ and $m(\Phi)$ evaluated at the local maximum of the potential, ψ has the same mass on the soliton as on the original D25-brane. This shows that the spectrum of the soliton is inherited from the D25-brane, and so can be identified with the spectrum of a D23-brane including all massive string states.

4. Non-commutative solitons on type-II D-branes

The construction of D-branes as non-commutative solitons in the bosonic string has an obvious extension to type-II superstring theory. Type-IIA theory contains non-BPS Dp-branes for p odd and IIB theory has non-BPS Dp-branes for p even [18, 26, 27, 28] which have a natural interpretation as sphalerons [29]. Solitons in level truncated open superstring field theory have been studied in [7].

To be concrete consider the non-BPS space-filling $D9$-brane of IIA theory. As before, we turn on a B-field in two dimensions and consider the limit of large $\alpha' B$. The analog of equation (2.8) is

$$\frac{C}{G_s}\int d^{10}x\,\sqrt{G}\left(\frac{1}{2}f(t)G^{\mu\nu}\partial_\mu t\partial_\nu t + \cdots - V(t)\right)\,, \tag{4.1}$$

with

$$C = g_s T_{9A}\,. \tag{4.2}$$

T_{9A} is the tension of the type-IIA D9-brane.

As before, the tachyon potential is not known exactly, but it is known to have a reflection symmetry and the height of the potential follows from the identification of the global minimum with the closed string vacuum. We choose the global minimum to be at $t = 0$ for easier comparison with our previous results. The reflection symmetry then acts by $(t - t_*) \to -(t - t_*)$. A schematic potential with these properties is given in figure 2.

Repeating the analysis done in the bosonic string shows that the solution

$$t = t_*\phi_0 \tag{4.3}$$

has the same tension as a D7-brane. By turning on constant B-fields in an even number of dimensions we construct the rest of the non-BPS branes of IIA. Note that in contrast to the bosonic string, here we obtain all non-BPS D-branes through this construction since in type-II theory the codimension of these branes differs by an even integer.

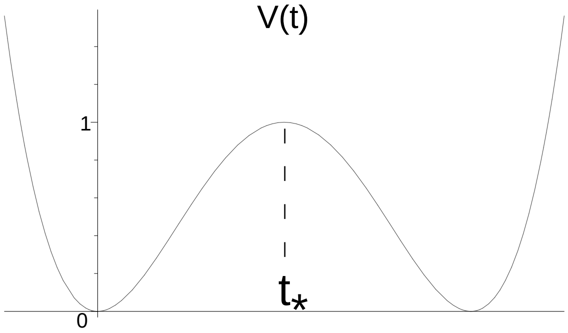

Figure 2: The open superstring tachyon potential.

This solution is unstable as in the previous analysis with the instability reflecting the presence of a tachyon of the correct mass on the D7-brane. Multiple D-branes can be incorporated as before, and the analysis of gauge field couplings is essentially the same as the discussion in the previous section.

We can apply the reflection symmetry to obtain the reflected solution[3]

$$t = t_*(2 - \phi_0)\,. \tag{4.4}$$

This reflected solution represents a physically distinct configuration; it is asymptotic to a solution whose ten-form RR field strength differs by one unit from (4.3) [29].

[3]Note that this solution is not of the canonical form described in [17] of a critical point of the potential times a projection operator. If $V'(t) = t\prod_i(t - \lambda_i)$ then $t = \lambda_k(P + 1)$ with $P^2 = P$ is a solution if $\lambda_k = \lambda_i/2$ for some i.

We can also construct an unexpected solution (and its Z_2 image) since the potential has an additional stationary point at $t = 2t_*$:

$$t = 2t_* \phi_0 \, . \tag{4.5}$$

Since $V(2t_*)$ vanishes, this soliton has vanishing tension in the limit of infinite non-commutativity! It is easy to see from the considerations in [17] that this object is stable. If we apply this construction in 10 euclidean dimensions we would seem to obtain a zero action instanton. The presence of such an object would clearly have dramatic consequences.

One way to understand the solution (4.5) is to consider a level k solution

$$t_k = 2t_*(\phi_0 + \phi_1 + \cdots + \phi_{k-1}) \tag{4.6}$$

in the limit $k \to \infty$ as in the discussion in section 2.4. This gives the solution $t_\infty = 2t_*$ which is simply the Z_2 image of the closed string vacuum at $t = 0$. In the same way that adding up an infinite number of D-23 branes gave a D25-brane, here adding up an infinite number of these tensionless 7-branes gives the other closed string vacuum.

It may well be that at finite $\alpha' B$ this solution develops a non-zero tension representing the fact that once derivatives are included it costs non-zero action to move from one vacuum to the other. A subleading tension of order $1/(g_s \alpha' B)$ would also have interesting consequences since there would then be a one-parameter family of non-perturbative objects with variable tension. Since asymptotically one is in the closed string vacuum and B can be gauged to zero, the value of B is not a closed string modulus, but rather some modulus of the solution. In principle the tension could be non-zero only at one string loop, but this seems unlikely since it would modify the perturbative structure of string theory. More work is clearly needed to understand this mysterious object.

The other unstable D-brane system of interest in type-II string theory is a Dp-$\overline{\mathrm{D}}p$ system of BPS Dp-branes. These objects have a complex tachyon with a "Mexican hat" potential given by rotating figure 2 about a vertical axis at $t = t_*$. There are also two U(1) gauge fields, A^+ and A^-, coming from the D-brane and anti D-brane respectively, and the tachyon carries charge one under the relative U(1) with gauge field $A^+ - A^-$. Both of the U(1)'s become non-commutative in the presence of a non-zero B-field with the tachyon transforming in the bi-fundamental representation.

For topologically trivial solutions we can use the relative U(1) to make the tachyon field t real. The above solutions are then also solutions to this system. Since t is real, the tachyon configuration does not act as a source for the relative U(1) gauge field and so the solutions carry no net lower D-brane charge. The coefficient in front of the action is now $2T_p$, so we can interpret the analogue of the first solution (4.3) as an unstable $D(p-2) - \overline{D(p-2)}$ brane system with energy $2T_{p-2}$.

The analogue of the second solution (4.5) is apparently a tensionless bound state of the $D(p-2)-\overline{D(p-2)}$ system.

BPS $D(p-2)$-branes can be constructed as vortices in the complex tachyon field [18]. One can find an exact vortex solution using a two-dimensional effective action which includes the tachyon field and the $\mathrm{U}(1)\times\mathrm{U}(1)$ gauge fields[4]

The action is

$$S=\int dz d\bar{z}\left(\overline{D_\mu t}D^\mu t-\frac{1}{4}F^+_{\mu\nu}F^{+\mu\nu}-\frac{1}{4}F^-_{\mu\nu}F^{-\mu\nu}-V(t,\bar{t})\right). \tag{4.7}$$

The gauge field strengths have the canonical form and the covariant derivatives are given by

$$D_\mu t=\partial_\mu t+i(A^+_\mu t-tA^-_\mu) \tag{4.8}$$

and

$$\overline{D_\mu t}=\partial_\mu\bar{t}-i(\bar{t}A^+_\mu-A^-_\mu\bar{t})\,. \tag{4.9}$$

In the limit of large non-commutativity, where we can ignore derivative terms, the equations of motion are

$$\begin{aligned}[][A^{+\nu},[A^+_\nu,A^+_\mu]] &= A^+_\mu t\bar{t}-tA^-_\mu\bar{t}+t\bar{t}A^+_\mu-tA^-_\mu\bar{t}\\ [A^{-\nu},[A^-_\nu,A^-_\mu]] &= A^-_\mu\bar{t}t-\bar{t}A^+_\mu t+\bar{t}tA^-_\mu-\bar{t}A^+_\mu t\\ -A^+_\mu A^{+\mu}t+2A^{\mu+}tA^-_\mu-tA^{-\mu}A^-_\mu &= -\alpha tV'-\beta V't\,.\end{aligned} \tag{4.10}$$

The values of α,β depend on the ordering prescription for V.

These equations are solved by

$$\begin{aligned} t &= t_*\sum_{i=0}^{\infty}\phi_{i,i+1}\\ A^+_z &= A^+_{\bar{z}}=t\bar{t}=1\\ A^-_z &= A^-_{\bar{z}}=\bar{t}t=1-\phi_0\,.\end{aligned} \tag{4.11}$$

Note that the field strengths of both gauge fields vanish since $A^\pm_z$ are hermitean and so $F_{z\bar{z}}\sim[A_z,A_{\bar{z}}]=0$. Similarly, $D_\mu D^\mu t$ vanishes, so to get a solution we can choose the ordering in V to be $V(\bar{t}t-1)$. This gives $\alpha=1,\beta=0$ and we have a solution using $tV'\propto t\phi_0=0$.

In the commutative case the vortex carries a non-zero D7 charge which arises from the coupling of the RR potential to the relative U(1) gauge field strength, $\int C_8\wedge(F^+-F^-)$. In the large non-commutativity limit we have found $F^+-F^-=0$, so one would naively think that there is no induced $D7$ charge. This is incorrect.

[4] The previous version of this paper had a crucial minus sign error in the equations of motion; the vortex solution presented in [30] was an important guide in correcting this error.

The couplings of RR fields to gauge and tachyon fields take an elegant form worked out in [31]. The relevant contribution is

$$\int C \wedge d\,\mathrm{Tr}\left(t \wedge \overline{Dt}\right). \tag{4.12}$$

Substituting (4.11) one finds that this term gives the correct induced $D7$ charge. To show this, it is important to keep the spatial derivative term in $D_\mu t$.

5. Fundamental strings as electric flux tubes

After tachyon condensation the open string degrees of freedom are confined and excitations of the theory are closed strings. We would like to describe these excitations as solitons. To this end we construct in the following an electric flux tube with the tension of the fundamental string. The classical confinement of electric flux found here is similar in spirit to that discussed in [32], but distinct from that discussed in [11, 10].

Naively, a gauge theory action of the type (3.6) gives, in the strong coupling limit $h(t)G_s^{-1} \to 0$, very heavy electric flux tubes — one expects a tension of order $h^{-1}G_s$. However, with the nonlinearity inherent in the Born-Infeld action, we will see that the energy cost of a flux quantum saturates, and the flux tube remains light in the limit.

5.1 The flux tube and its tension

We need the effective action for the tachyon and the gauge fields for configurations carrying electric flux in a particular direction, say x^1. The task is simplified with the introduction of a large B-field in all 24 transverse directions x^i, $i = 2, \ldots, 25$. Then derivatives along these directions are negligible. Furthermore, it is sufficient (for now) to consider gauge field configurations that are constant in time and along the flux tube. The tachyon potential in the presence of *constant* background fields is known from a theorem of Sen [33, 2]: the potential is universal up to tachyon independent deformations of the overall metric. Thus, in our context, the action is of the Born-Infeld type

$$S = -\int d^{26}x\ V(t)\sqrt{-\det[\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}]}\,. \tag{5.1}$$

(Other recent discussions of the action include [34, 35]). Coordinates were chosen so $G_{\mu\nu} = \eta_{\mu\nu}$, and $\star$ products are implied. In this section we absorb the overall tension of the D25-brane in the potential $V(t_*)$. It is sufficient to retain just one component of the gauge field, i.e.

$$S = -\int d^{26}x\ V(t)\sqrt{1 - (2\pi\alpha' \dot{A}_1)^2}\,. \tag{5.2}$$

The analysis will be simplest in the canonical formalism. We therefore compute the momentum conjugate to the gauge field

$$\mathcal{E} = \frac{V(t)(2\pi\alpha')^2 \dot{A}_1}{\sqrt{1-(2\pi\alpha' \dot{A}_1)^2}}\,. \tag{5.3}$$

Quantization of the electric flux $\mathcal{E}$ plays an important role in our discussion. In the large B limit $\mathcal{E}$ is the electric flux of a $1+1$ dimensional $\mathrm{U}(\infty)$ gauge theory. Flux quantization in 1+1 Yang-Mills theory is analyzed in [36], and we can apply a similar analysis here. Electric flux receives contributions from two sources: from matter and gauge field charge fluctuations, and from charges at infinity. In the present context these charges have values corresponding to the endpoints of open strings. It follows that the electric flux in each $\mathrm{U}(1)$ subgroup of $\mathrm{U}(\infty)$ is quantized, and that it can be thought of roughly as the number of open strings (signed depending on orientation) ending on the corresponding D23-brane. More precisely, in a diagonal basis the eigenstates of $\mathcal{E}$ are

$$\mathcal{E} = \frac{1}{(2\pi\theta)^{12}} \sum_{k=0}^{\infty} n_k \phi_k\,, \qquad n_k \in \mathbb{Z}. \tag{5.4}$$

From now on we will drop the overall factors of $(2\pi\theta)^{-12}$; they can be absorbed into field redefinitions. The overall $\mathrm{U}(1)$ flux,

$$\int d^{24}x\, \mathcal{E} = \sum_{k=0}^{\infty} n_k \equiv N \tag{5.5}$$

is a conserved quantity and can be identified with the total number of fundamental strings in the state. The individual eigenvalues n_k are not conserved, and the general quantum state will correspond to a superposition of different n_k. In a complete quantum mechanical treatment it would be important to include the effect of transitions among different flux states,[5] but we will be less ambitious and consider the properties of a single flux state of the form (5.4).

We consider the hamiltonian relevant to finding the configuration which minimizes the energy in a given total flux sector,

$$H = \int d^{25}x \left[\sqrt{V(t)^2 + \frac{\mathcal{E}^2}{(2\pi\alpha')^2}} + \lambda\partial_1\mathcal{E}\right] - \lambda N\,, \tag{5.6}$$

[5]One might expect such fluctuations to be rather large, since the effective coupling $1/V(t)$ in (5.2) (we have absorbed a factor of $1/g_s$ in the tachyon potential) is quite large as the tachyon field approaches the closed string vacuum $V(t) \to 0$. It would be interesting if the fluctuations could be related to the usual divergent vacuum fluctuations in the spatial position of a perturbative closed string. We thank S. Shenker and A. Lawrence for discussions on this point.

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where λ is a Lagrange multiplier enforcing the condition that we consider configurations with N flux quanta. The corresponding equations of motion are

$$\frac{\delta H}{\delta t} = \frac{V(t)V'(t)}{\sqrt{V(t)^2 + \mathcal{E}^2/(2\pi\alpha')^2}} = 0\,, \tag{5.7}$$

$$\frac{\delta H}{\delta \mathcal{E}} = \left[\frac{\mathcal{E}}{(2\pi\alpha')^2\sqrt{V(t)^2 + \mathcal{E}^2/(2\pi\alpha')^2}} + \lambda\right] = 0\,. \tag{5.8}$$

We want solutions that exist in the vacuum after tachyon condensation, so $V(t)$ should vanish asymptotically. We therefore try solutions of the form

$$t = t_0\phi\,, \tag{5.9}$$

where ϕ is one of the functions satisfying $\phi \star \phi = \phi$. (5.7) is solved by taking t_0 to be an extremum of V. Taking $t = t_*$, (5.8) is solved by

$$\mathcal{E} = e\phi\,, \tag{5.10}$$

and the flux quantization condition (5.5) determines $e = N/k$ for ϕ at level k. Eq. (5.8) also determines the Lagrange multiplier λ, but this is not needed in the subsequent discussion. Inserting the solution back into the hamiltonian (5.6) the tension is found to be

$$T = \sqrt{k^2 V(t_*)^2 + \frac{N^2}{(2\pi\alpha')^2}} = \frac{1}{2\pi\alpha'}\sqrt{\frac{k^2}{g_s^2} + N^2}\,. \tag{5.11}$$

This is as expected for an (N,k) type string.

An even simpler solution is found by taking $t = 0$. In this case $V(0) = 0$ so (5.8) places no constraint on the form of $\mathcal{E}$, although it does determine λ. $\mathcal{E}$ can take any form (5.4) consistent with the flux quantization condition (5.5). The tension of such a solution is

$$T = \sqrt{V(0)^2 + e^2/(2\pi\alpha')^2} = \frac{N}{2\pi\alpha'}\,, \tag{5.12}$$

which is the result expected for N fundamental strings.

The way we derived it, the tension was essentially guaranteed to come out right. The nontrivial part was the magic of non-commutativity, which allows one to find a *localized* solution to equations which would otherwise be difficult to analyze.

The action that we started with (5.1) is proportional to $1/g_s$ (absorbed in $V(t)$). As mentioned above, it is therefore surprising that the flux tube solution (5.12) has energy of $\mathcal{O}(g_s^0)$. This is possible because, in the nonlinear Born-Infeld hamiltonian (5.6), the coefficient of the electric field term is independent of $V(t)$, whereas it would have a coefficient $1/V$ in a Yang-Mills type action. Alternatively, the usual expansion of the Born-Infeld action in powers of the field strength is inappropriate; one is in a 'relativistic' limit of the field dynamics, with V playing the role of mass.

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5.2 Fluctuations

We would like to consider also excitations of the fundamental string. This is more demanding because generally the action (5.1) receives corrections with unsuppressed derivatives in the longitudinal direction x^1. However, the action is still applicable for fluctuations with large wavelength $\sqrt{\alpha'}F'_{\mu\nu} \ll F_{\mu\nu}$ (prime denotes derivatives along x^1). Note that this condition still allows for amplitudes of order string scale $\alpha' F_{\mu\nu} \sim 1$ so the nonlinear terms in the Born-Infeld action do contain valid information. As mentioned earlier, a true quantum state corresponding to a fundamental consists of a superposition of solutions with a given total electric flux. However, we will consider the simpler exercise of analyzing fluctuations around a single solution with fixed n_k, and leave the more complete treatment for future work.

Again, our considerations will be simplest in the canonical formalism. To find the hamiltonian, we expand the determinant in (5.1) and write the action

$$S = -\int d^{26}x\ V(t)\sqrt{\det(\mathbb{1}+\mathbf{F})(1-F_{0\alpha}M^{\alpha\beta}F_{0\beta})}\,, \tag{5.13}$$

where

$$M^{\alpha\beta} = \left(\frac{\mathbb{1}}{\mathbb{1}+\mathbf{F}}\right)^{\alpha\beta}_{\text{sym}}\,. \tag{5.14}$$

Here $\alpha, \beta, \ldots$ are purely spatial indices and the matrices $\mathbb{1}, \mathbf{F}$ have components $\delta_{\alpha\beta}, F_{\alpha\beta}$. In this subsection we take $2\pi\alpha' = 1$ to simplify the formulae; these factors are easily restored by comparing with the previous computation. The canonical momenta are

$$\mathcal{E}^\alpha = \frac{V(t)M^{\alpha\beta}F_{0\beta}}{\sqrt{1-F_{0\alpha}M^{\alpha\beta}F_{0\beta}}}\,, \tag{5.15}$$

and the hamiltonian becomes

$$H = \int d^{25}x\ \left(\mathcal{E}^\alpha \dot{A}_\alpha - \mathcal{L}\right) = \int d^{25}x\ \left[\sqrt{\mathcal{E}^\alpha M_{\alpha\beta}\mathcal{E}^\beta + V(t)^2\det(\mathbb{1}+\mathbf{F})} + A_0\nabla_\alpha\mathcal{E}^\alpha\right], \tag{5.16}$$

where

$$M_{\alpha\beta} = \delta_{\alpha\beta} - F_{\alpha\gamma}F^{\gamma}{}_{\beta}\,. \tag{5.17}$$

The equation of motion for A_0 is the Gauss law constraint

$$\nabla_\alpha \mathcal{E}^\alpha = 0\,. \tag{5.18}$$

Hereafter it is convenient to choose the gauge $A_0 = 0$.

Derivatives in the transverse directions are negligible, so transverse field strengths simplify: $F_{ij} = 0$ and $F_{1i} = A'_i$. The hamiltonian can therefore be written

$$H = \int d^{25}x\sqrt{(\mathcal{E}^1)^2(1+(\vec{A}')^2) + \vec{\mathcal{E}}^2 + (\vec{\mathcal{E}}\cdot\vec{A}')^2 + V(t)^2(1+(\vec{A}')^2)}\,. \tag{5.19}$$

The vector notation applies to the transverse directions x^i, $i = 2, \ldots, 25$. Considering just the fundamental string we can take $V(t) = 0$ from here onwards.

Fluctuating strings are described as flux tube solutions based on functions satisfying $\phi \star \phi = \phi$, as before, but with the origin displaced in a manner depending on the coordinates along the string

$$\phi = \phi(x^i - f^i(x^0, x^1))\,. \tag{5.20}$$

A suitable *ansatz* is

$$\mathcal{E}_1 = e_1 \phi\,, \qquad \vec{\mathcal{E}} = \vec{e}\phi\,, \qquad \vec{A}' = \vec{a}'\phi\,. \tag{5.21}$$

The coefficient e_1 will generally be N/k, but in this section we consider a single string, so $e_1 = 1$. The functions $\vec{e}, \vec{a}$ depend on the string coordinates $x^a = x^0, x^1$ but not on the transverse coordinates x^i.

We want to find the effective dynamics of the variables f^i. It is not sufficient to insert our *ansatz* in the hamiltonian (5.19) because that would not ensure that the equations of motion for other fields are satisfied. The correct procedure is known as hamiltonian reduction (it is described in detail in a closely related context in [37]). By solving the constraints, the hamiltonian is expressed in terms of the reduced variables $\vec{f}$ and their conjugate momenta. In the present context the important constraint is Gauss' law (5.18). Inserting our *ansatz* we find

$$-e_1 \frac{\partial \phi}{\partial x^i} \frac{\partial f^i}{\partial x^1} + e^i \frac{\partial \phi}{\partial x^i} = 0\,. \tag{5.22}$$

Recalling that $e_1 = 1$, a simple solution is

$$\vec{f}' = \vec{e}\,. \tag{5.23}$$

We need to find the momentum conjugate to $\vec{f}'$. The fields $\vec{\mathcal{E}}$ and $\vec{A}$ are canonically conjugate in the original theory. This implies that $\vec{e}$ and $\vec{a}$ are canonically conjugate in the reduced theory. From (5.23) we therefore find that the momenta conjugate to $\vec{f}'$ are $\vec{\pi} \equiv \vec{a}'$.[6] We conclude that the reduced hamiltonian

$$H = \int dx^1 \, \sqrt{1 + \vec{\pi}^2 + (\vec{f}')^2 + (\vec{\pi} \cdot \vec{f}')^2}\,, \tag{5.24}$$

describes the fluctuations of the flux tube. The dynamics of transverse modes was "integrated out" in the hamiltonian reduction.

The significance of this hamiltonian is best exhibited in the lagrangian formalism. We therefore compute the time derivative using Hamilton's equations

$$\dot{\vec{f}} = \frac{\delta H}{\delta \vec{e}} = \frac{\vec{\pi} + \vec{f}'(\vec{f}' \cdot \vec{\pi})}{\sqrt{1 + \vec{\pi}^2 + (\vec{f}')^2 + (\vec{\pi} \cdot \vec{f}')^2}}\,, \tag{5.25}$$

[6] Up to a factor of the wavenumber, when we expand in the usual oscillator basis.

and find

$$L = \int d^2x \; \vec{\pi}\cdot\dot{\vec{f}} - \int dx^0 \; H = -\int d^2x \sqrt{(1-(\dot{\vec{f}})^2)(1+(\vec{f}')^2)+(\dot{\vec{f}}\cdot\vec{f}')^2}\,. \quad (5.26)$$

This should be compared with the standard Nambu-Goto action

$$L_{NG} = -\int d^2x \sqrt{-\det[\partial_a X_\mu \partial^a X^\mu]} = -\int d^2x \sqrt{(\dot{\vec{X}})^2(\vec{X}')^2-(\dot{\vec{X}}\cdot\vec{X}')^2}\,. \quad (5.27)$$

In the static gauge $X^\mu = (x^0, x^1, f^i)$ used here this reduces precisely to (5.26). The effective action for the fluctuations of the flux tube is therefore the Nambu-Goto action!

The next step is to quantize the fluctuations of the flux tube. Their action is the Nambu-Goto action so the result is that of a closed fundamental string. We have justified the lagrangian (5.1) only for large semi-classical waves so it is in this regime that the spectrum of the flux tube should be compared with that of fundamental strings. Our computation demonstrates a perfect agreement in the allowed energies and their degeneracies.

It is tantalizing that, if we extend this reasoning beyond its apparent regime of validity, we find the entire fundamental string spectrum as simple excitations in the open string field theory. This would include *very* light modes — such as the massless graviton, and even the closed string tachyon. Usually this kind of identification would be impossible *in principle*: the quantum fluctuations of a soliton are collective excitations rather than fundamental objects, because the soliton itself is made out of more basic constituents. We are better off here because, after tachyon condensation, the flux tubes are the lightest objects in the theory and therefore subject to quantization. Although this removes the objection of principle, we presently have no justification to trust our result for light modes.

5.3 Multiple strings and interactions

One can also consider electric fluxes lying in U(k) subgroups of the U(∞) gauge group, leading to additional solutions. These parallel closely the construction of Matrix string theory [38]. With x^1 compactified on a large circle of radius R, one can describe multiply wrapped strings via the holonomy of the gauge field that twists the eigenvalues of the U(k) electric field into a 'long string' or 'slinky'. One can follow the arguments given in [38] leading to the identification of the interactions of such a string with the effective twist operator that arises when SU(2) symmetry is restored by two strands of the Matrix string coming together in the transverse space. Thus we see that the flux tubes at least qualitatively interact in the proper way; it would be interesting to see if at least some perturbative string amplitudes can be reproduced within the non-commutative framework (for instance, tree level amplitudes that just exchange winding among these macroscopic strings). A potential difficulty in this exercise is that the original string coupling constant g_s appears only through the tachyon potential $V(t)$ in (5.19); after tachyon condensation $V \to 0$, thus it seems that the electric flux lines do not know about the original string coupling. One does not a priori know why the interaction strength of the flux lines is $\mathcal{O}(g_s)$ and not $\mathcal{O}(1)$.

One might also be concerned that there appear to be two separate descriptions of a k times wound fundamental string: Namely k units of flux in a U(1) solution, as well as k units of flux wound into a 'slinky' in U(k). In Matrix string theory, these two carry different quantum numbers; the rank of the gauge group corresponds to the number of bits of discrete light cone momentum of the Matrix string, while the electric flux represents D0-brane charge that the string is bound to. In the present context, both of these objects are embedded in the same U(∞) gauge theory, and simply represent different spatial distributions of the flux lines; there is no apparent reason why the the path integral over the configuration space of the flux lines in the strongly coupled gauge theory will not smoothly evolve these two flux configurations into one another.

To summarize, we find macroscopic fundamental strings appearing as light electric flux excitations of the open string field theory, after the tachyon is condensed to form the closed string vacuum. The tension of the flux tubes is just right, and the fluxes join and split in the manner familiar from string perturbation theory (although it is not at present understood whether their interaction strength is related to the string coupling g_s).

6. Comments and questions

It is remarkable that turning on a large B-field simplifies the structure of open string field theory in such a dramatic way that one can construct exact D-brane and fundamental string solutions. The full force of this construction relies heavily on Sen's conjecture that tachyon condensation represents the closed string vacuum. We use this conjecture to determine the height of the tachyon potential, but also to argue that at the end of the construction one can gauge B to zero far from the solution so that one is discussing D-branes and fundamental string in the usual closed string vacuum with $B = 0$.

Similarities to Matrix theory have been a persistent thread running through our discussion. In both cases a large dimensionless parameter is introduced — $\alpha' B$ for the non-commutative limit, and the boost rapidity in Matrix theory — upon which quantities of interest do not depend, yet whose introduction facilitates calculations. The large B-field of non-commutative geometry induces a macroscopic density of lower-dimensional D-branes, while tachyon condensation removes the higher-dimensional D-brane and makes the effective gauge coupling large in regions described by the closed string vacuum. Thus the ingredients of the Matrix theory limit are present; furthermore, the scaling limit of [16] is the analogue of the Maldacena scaling limit [39] ($\alpha' \to 0$, with the energies of stretched open strings held fixed). It would be interesting to make the relation between these two circles of ideas more precise.

Our work raises a number of other interesting questions:

1. Is it possible to also construct the NS 5-brane (or 21-brane in the bosonic string) using these methods?[7]

2. How does one understand the freezing out of the open string degrees of freedom and the ability to gauge B to zero in the closed string vacuum, directly in string field theory?

3. What is the interpretation of the very light solitons (massless to leading order in $1/\alpha' B$) found in the superstring?

4. What are the leading $1/\alpha' B$ corrections to the results presented here?

5. What is the coupling strength of the fundamental string constructed in section 4?

We hope to address some of these questions in future work.

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Note added. As this work was being typed we became aware of work by K. Dasgupta, S. Mukhi, and G. Rajesh [40] which overlaps our discussion of non-commutative solitons in type-II string theory. We thank them for bringing this work to our attention.

References

[1] A. Sen, *Descent relations among bosonic D-branes*, *Int. J. Mod. Phys.* **A 14** (1999) 4061 [hep-th/9902105].

[7] The analogy with Matrix theory suggests that the problem is similar in nature to the construction of the transverse Matrix fivebrane. In particular, the energy density of the object scales as g_s^{-2}, whereas classical open string field configurations have energy densities scaling as g_s^{-1}.

[2] A. Sen, *Universality of the tachyon potential*, *J. High Energy Phys.* **12** (1999) 027 [hep-th/9911116].

[3] A. Sen and B. Zwiebach, *Tachyon condensation in string field theory*, *J. High Energy Phys.* **03** (2000) 002 [hep-th/9912249].

[4] J.A. Harvey and P. Kraus, *D-branes as unstable lumps in bosonic open string field theory*, *J. High Energy Phys.* **04** (2000) 012 [hep-th/0002117].

[5] R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, *Lumps and p-branes in open string field theory*, *Phys. Lett.* **B 482** (2000) 249 [hep-th/0003031].

[6] V.A. Kostelecky and S. Samuel, *On a nonperturbative vacuum for the open bosonic string*, *Nucl. Phys.* **B 336** (1990) 263.

[7] N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, hep-th/0002211.

[8] J.A. Harvey, D. Kutasov and E.J. Martinec, *On the relevance of tachyons*, hep-th/0003101.

[9] A. Sen, *Supersymmetric world-volume action for non-BPS D-branes*, *J. High Energy Phys.* **10** (1999) 008 [hep-th/9909062].

[10] O. Bergman, K. Hori and P. Yi, *Confinement on the brane*, *Nucl. Phys.* **B 580** (2000) 289 [hep-th/0002223].

[11] P. Yi, *Membranes from five-branes and fundamental strings from Dp-branes*, *Nucl. Phys.* **B 550** (1999) 214 [hep-th/9901159].

[12] C.G. Callan Jr. and J.M. Maldacena, *Brane dynamics from the Born-Infeld action*, *Nucl. Phys.* **B 513** (1998) 198 [hep-th/9708147].

[13] P.S. Howe, N.D. Lambert and P.C. West, *The self-dual string soliton*, *Nucl. Phys.* **B 515** (1998) 203 [hep-th/9709014].

[14] G.W. Gibbons, *Born-Infeld particles and Dirichlet p-branes*, *Nucl. Phys.* **B 514** (1998) 603 [hep-th/9709027].

[15] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [hep-th/9711162].

[16] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[17] R. Gopakumar, S. Minwalla and A. Strominger, *Noncommutative solitons*, *J. High Energy Phys.* **05** (2000) 020 [hep-th/0003160].

[18] A. Sen, *Stable non-BPS bound states of BPS D-branes*, *J. High Energy Phys.* **08** (1998) 010 [hep-th/9805019]; SO(32) *spinors of type-I and other solitons on brane- antibrane pair*, *J. High Energy Phys.* **09** (1998) 023 [hep-th/9808141]; *Type-I D-particle and its interactions*, *J. High Energy Phys.* **10** (1998) 021 [hep-th/9809111]; *Non-BPS states and branes in string theory*, hep-th/9904207, and references therein.

[19] A. Dhar, G. Mandal and S.R. Wadia, *String field theory of two-dimensional QCD: a realization of W_∞ algebra*, *Phys. Lett.* **B 329** (1994) 15 [hep-th/9403050]; *Nonrelativistic fermions, coadjoint orbits of W_∞ and string field theory at $c = 1$*, *Mod. Phys. Lett.* **A 7** (1992) 3129 [hep-th/9207011].

[20] N. Moeller and W. Taylor, *Level truncation and the tachyon in open bosonic string field theory*, hep-th/0002237.

[21] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[22] B. de Wit, J. Hoppe and H. Nicolai, *On the quantum mechanics of supermembranes*, *Nucl. Phys.* **B 305** (1988) 545.

[23] M.R. Garousi, *Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action*, hep-th/0003122.

[24] N. Ishibashi, *A relation between commutative and noncommutative descriptions of D-branes*, hep-th/9909176.

[25] J.X. Lu and S. Roy, *(p+1)-dimensional noncommutative Yang-Mills and D$(p-2)$ branes*, *Nucl. Phys.* **B 579** (2000) 229 [hep-th/9912165].

[26] O. Bergman and M.R. Gaberdiel, *Stable non-BPS D-particles*, *Phys. Lett.* **B 441** (1998) 133 [hep-th/9806155].

[27] A. Sen, *BPS D-branes on non-supersymmetric cycles*, *J. High Energy Phys.* **12** (1998) 021 [hep-th/9812031].

[28] P. Hořava, *Type-IIA D-branes, K-theory and matrix theory*, *Adv. Theor. Math. Phys.* **2** (1999) 1373 [hep-th/9812135].

[29] J.A. Harvey, P. Hořava and P. Kraus, *D-sphalerons and the topology of string configuration space*, *J. High Energy Phys.* **03** (2000) 021 [hep-th/0001143].

[30] E. Witten, *Noncommutative tachyons and string field theory*, hep-th/0006071.

[31] C. Kennedy and A. Wilkins, *Ramond-Ramond couplings on brane-antibrane systems*, *Phys. Lett.* **B 464** (1999) 206 [hep-th/9905195].

[32] J. Kogut and L. Susskind, *Vacuum polarization and the absence of free quarks in four-dimensions*, *Phys. Rev.* **D 9** (1974) 3501.

[33] A. Sen, *Supersymmetric world-volume action for non-BPS D-branes*, *J. High Energy Phys.* **10** (1999) 008 [hep-th/9909062].

[34] E.A. Bergshoeff, M. de Roo, T.C. de Wit, E. Eyras and S. Panda, *T-duality and actions for non-BPS D-branes*, *J. High Energy Phys.* **05** (2000) 009 [hep-th/0003221].

[35] J. Kluson, *Proposal for non-BPS D-brane action*, hep-th/0004106.

[36] E. Witten, *Theta vacua in two-dimensional quantum chromodynamics*, *Nuovo Cim.* **A51** (1979) 325.

[37] F. Larsen and F. Wilczek, *Classical hair in string theory, 1. General formulation*, *Nucl. Phys.* **B 475** (1996) 627 [hep-th/9604134].

[38] L. Motl, *Proposals on nonperturbative superstring interactions*, hep-th/9701025;
T. Banks and N. Seiberg, *Strings from matrices*, *Nucl. Phys.* **B 497** (1997) 41 [hep-th/9702187];
R. Dijkgraaf, E. Verlinde and H. Verlinde, *Matrix string theory*, *Nucl. Phys.* **B 500** (1997) 43 [hep-th/9703030].

[39] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[40] K. Dasgupta, S. Mukhi and G. Rajesh, *Noncommutative tachyons*, *J. High Energy Phys.* **06** (2000) 022 [hep-th/0005006].